Applications

1. 12 cm
2. a. The 12th triangle has leg lengths 1 unit and $\sqrt{12}$ units and hypotenuse length $\sqrt{13}$ units. The 13th triangle has leg lengths 1 unit and $\sqrt{13}$ units and hypotenuse length $\sqrt{14}$ units. The 14th triangle has leg lengths 1 unit and $\sqrt{14}$ units and hypotenuse length $\sqrt{15}$ units.

   b. $\frac{1}{2}$ units^2, $\frac{1}{2}\sqrt{2}$ units^2, $\frac{1}{2}\sqrt{3}$ units^2, $\frac{1}{2}\sqrt{4}$ units^2, $\frac{1}{2}\sqrt{5}$ units^2. The number under the square root sign increases by 1 for every new triangle. Or, the area of the *n*th triangle is $\frac{1}{2}\sqrt{n}$.

   c. 5 is the square root of 25. So, the hypotenuse length of the 24th triangle is 5 units.
3. $\frac{5}{8} = 0.625$; terminating
4. $\frac{3}{5} = 0.6$; terminating
5. $\frac{1}{6} = 0.1666$ …; 6 repeats
6. $\frac{4}{99} = 0.04040404$ …; 0.4 repeats
7. $\frac{43}{10} = 4.3$; terminating
8. Possible answer: $\frac{1{,}875}{10{,}000}$, or $\frac{3}{16}$
9. Possible answer: $5\frac{1}{8}$
10. Possible answer: $43\frac{6}{10}$
11. a. $\frac{1}{99} = 0.010101$ …, $\frac{2}{99} = 0.020202$ …, $\frac{3}{99} = 0.030303$ … A fraction with a denominator of 99 is equal to a repeating decimal. For numerators less than 99, the repeating part has two digits: either a 0 followed by the number in the numerator if that number is less than 10 or the number in the numerator if that number is greater than 10.

    b. $\frac{51}{99} = 0.51515151$ …
12. a. $\frac{1}{999} = 0.001001001$ …, $\frac{2}{999} =$ 0.002002002 …, $\frac{3}{999} =$ 0.003003003 … A fraction with a denominator of 999 is equal to a repeating decimal. For numerators less than 999, the repeating part has three digits: two 0s followed by the number in the numerator if that number is less than 10; one 0 followed by the number in the numerator if that number is greater than 10 and less than 100; or the number in the numerator if that number is greater than 100.

    b. $\frac{1{,}000}{999} = 1.001001$ …
13. $\frac{1}{3}$
14. $\frac{5}{99}$
15. $\frac{45}{99}$, or $\frac{5}{11}$
16. $\frac{45}{999}$, or $\frac{15}{333}$
17. $10\frac{12}{99}$, or $\frac{334}{33}$
18. $3\frac{9}{9}$, or 4
19. There is an infinite number of answers. Possible answers: The decimal is between $\frac{1}{10}$ and $\frac{2}{10}$, between $\frac{10}{100}$ and $\frac{11}{100}$, and between $\frac{101}{1{,}000}$ and $\frac{102}{1{,}000}$.
20. There is an infinite number of answers. Possible answer: One decimal answer is 6.303003000300003 …
21. A possible solution from converting fractions to decimals: 0.1505005000500005…

    The following is a possible solution method considering fractions. Since $\left(\frac{1}{7}\right)^2 = \frac{1}{49}$ and $\left(\frac{1}{6}\right)^2 = \frac{1}{36}$, the square root of any number between $\frac{1}{49}$ and $\frac{1}{36}$ will be an irrational number between $\frac{1}{7}$ and $\frac{1}{6}$: for example, $\sqrt{\frac{1}{43}}$ or $\sqrt{0.025}$.
22. The bottom has sides of length 3 cm and 4 cm. Because $3^2 + 4^2 = 25$, the diagonal of the bottom has length $\sqrt{25}$ cm, or 5 cm. Using this as a leg of a right triangle with hypotenuse d, $d^2 = 5^2 + 12^2 = 169$, so $d = \sqrt{169} = 13.$ The length of diagonal d = 13 cm.
23. The bottom has sides of length 6 cm and 7 cm. Because $6^2 + 7^2 = 85$, the diagonal of the bottom has length $\sqrt{85}$ cm. Using this as a leg of a right triangle with hypotenuse d, $d^2 = \left(\sqrt{85}\right)^2 + \left(\sqrt{111}\right)^2 =$ 85 + 111 = 196, so $d = \sqrt{196} = 14.$ The length of diagonal d = 14 cm.

Connections

24. B

25. 6 and 7. $6^2 = 36$ and $7^2 = 49$. Because 39 is between 36 and 49, $\sqrt{39}$ is between 6 and 7.

26. 24 and 25. $24^2 = 576$ and $25^2 = 625$. Because 600 is between 576 and 625, $\sqrt{600}$ is between 24 and 25.

27. False. $0.06 \bullet 0.06 = 0.0036$

28. True. $1.1 \bullet 1.1 = 1.21$

29. False. $20 \bullet 20 = 400$

30. Right triangle; $5^2 + 7^2 = (\sqrt{74})^2$

31. Not a right triangle; $(\sqrt{2})^2 + (\sqrt{3})^2$ is not equal to 3^2.

32. 11; rational

33. 0.7; rational

34. approximately 3.9; irrational

35. approximately 31.6; irrational

36. **a.** Using the Pythagorean Theorem, $2^2 + h^2 = 29$, so the height h of the cone is 5 units.

b. The volume of the cylinder is $\pi(2)^2(5) = 20\pi$ units^3. So, the volume of the cone is $\frac{20\pi}{3}$ units^3.

37. **a.** 72 units^3. The volume of the cube is $6 \bullet 6 \bullet 6 = 216$ units^3. The volume of the pyramid is $\frac{1}{3}$ of the cube's volume, or 72 units^3.

b. $\frac{1}{3}x^3$. The cube has volume x^3. The volume of this pyramid is one third the volume of the cube, so it is $\frac{1}{3}x^3$.

Extensions

38. $12^2 - (\pi \times 6^2) \approx 30.9$ square units

39. $4 \times \left[6^2 - (\pi \times 3^2)\right] \approx 30.9$ square units

40. $9 \times \left[4^2 - (\pi \times 2^2)\right] \approx 30.9$ square units

41. **a.** 4. Using the commutative and associative properties, you can rewrite the equation as $(\sqrt{2})(\sqrt{2})(2) = (2)(2) = 4$

b. $\sqrt{2}$. Because you can rewrite 2 as $\sqrt{2} \times \sqrt{2}$, then $2 \div \sqrt{2} = \sqrt{2}$.

c. 2. Using the Distributive Property, you can rewrite and simplify the expression as $\sqrt{2} \times 1 + \sqrt{2} \times \sqrt{2} - \sqrt{2} =$ $\sqrt{2} + 2 - \sqrt{2} = 2$.

42. **a.** Using the Pythagorean Theorem, the length of half the edge of the base is 3 units, so the edge length of the base is 6 units. Therefore, the base area is 36 units^2.

b. The surface is made up of 4 congruent triangles plus a base. Each triangle has area $(\frac{1}{2})(6)(4) = 12$ units^2. So the surface area is $36 + 4(12) = 84$ units^2.

c. You can find the height of the pyramid from the right triangle with sides 3 units (half of the base edge) and 4 units (the slant height). You need to solve $3^2 + h^2 = 4^2$. h is $\sqrt{7}$ units, or about 2.65 units.

d. $(\frac{1}{3})(36)(2.65) = 31.8$ units^3.

43. **a.** 31.81 in.^{3}. Because the diameter is 4.5 in., the radius is 2.25 in. The height is 6 in., so the volume is $\frac{1}{3}\pi(2.25)^2(6) \approx 31.81$ in.^{3}.

b. 26π in.^{3}. $7^2 = r^2 + 6^2$, so $r^2 = \sqrt{13}$ in., or about 3.6 in. So the volume is $\frac{1}{3}\pi(\sqrt{13})^2(6) \approx 26\pi$ in.^{3}, or about 81.7 in.^{3}.