Applications

1. 22 ft. Because $25^2 - 15^2 = 400$, the tallest tree that can be braced is $\sqrt{400}$ ft, or 20 ft tall at the point of attachment. Adding 2 ft gives a total height of 22 ft. (**Note:** You can point out to students that this is a 3–4–5 Pythagorean Triple with a scale factor of 5.)

2. **a.** Because $500^2 + 600^2 = 610{,}000$, the length of the shortcut is $\sqrt{610{,}000} \approx$ 781 m.

   b. 1,100 – 781 ≈ 319 m

3. **a.** Option 1: 5.5 inches × 11.8 inches ($\sqrt{13^2 - 5.5^2} \approx 11.8$.)

   b. Option 2: 5.1 inches × 12.3 inches ($\sqrt{13.3^2 - 5.1^2} \approx 12.3$.)

4. The two shorter sides of the kite frame are each approximately 22.4 inches and the longer sides are each approximately 53.9 inches. So, the perimeter is approximately 152.6 inches. Ali is correct in saying they need 153 inches, which would give them less than one inch of extra string.

5. ≈ 105.5 ft. The leg along the bottom of the 30–60–90 triangle measures 58 ft. The hypotenuse (from Santos's eyes to the top of the tower) is twice as long, or 116 ft. Because $116^2 - 58^2 = 10{,}092$, the vertical leg measures $\sqrt{10{,}092} \approx 100.5$ ft. Adding the distance from the ground to Santos's eyes, the tower is about 105.5 ft tall.

6. **a.** *ABC*, *ADE*, and *AFG* are 30–60–90 triangles. The measure of angle *A*, which is in all three triangles, is 60°. Angles *ACB*, *AED*, and *AGF* all have measure 90°. You know that the sum of the measures of the angles of a triangle is 180°. So, the third angles of the three triangles—angles *ABC*, *ADE*, and *AFG*—must all have measure 30°. Triangles *ABC*, *ADE*, and *AFG* are all similar because if corresponding angles of a triangle have equal measure, then the triangles are similar.

   b. $\frac{BA}{AC} = \frac{4}{2} = \frac{2}{1}$. The length of *AC* is 2 units. Because triangle *ABC* is a 30–60–90 triangle, the length of *BA*, the hypotenuse, is twice the length of the side opposite the 30° angle, which is *AC*. Therefore, the length of *BA* is 4 units. The corresponding ratio for the other two triangles must be the same because the triangles are similar.

   c. $\frac{BC}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3}$. In a 30–60–90 triangle, the length of the side opposite the 60° angle is $\sqrt{3}$ times the length of the side opposite the 30° angle, which is *AC*. *AC* has length 2 units, so *BC* has length $2\sqrt{3}$ units. So, $\frac{BC}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3}$. The corresponding ratio for the other two triangles must be the same because the triangles are similar.

   d. $\frac{BC}{AB} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$. The corresponding ratio for the other two triangles must be the same because the triangles are similar.

   e. 16 units and $8\sqrt{3}$ units. In a 30–60–90 triangle, the length of the hypotenuse is twice the length of the side opposite the 30° angle, which is *AG*. *AG* has length 8 units, so *AF* has length 16 units. The length of the side opposite the 60° angle is $\sqrt{3}$ times the length of the side opposite the 30° angle. So, *GF* has length $8\sqrt{3}$ units.

7. **a.** Three. Triangles *KLM*, *KNL*, and *LNM*

   b. $18 + 6\sqrt{3}$ or ≈ 28.39 meters. All triangles in the diagram are 30–60–90 triangles. The hypotenuse of triangle *KLM* is 12 m (twice the shorter leg that is given). The longer leg is $6\sqrt{3}$, or $\sqrt{108} \approx 10.39$.

8. **a.** Corresponding angles have equal measure, so the triangles are similar.

b. Because the triangles are similar, you know that the side lengths of one triangle are multiplied by the same number, the scale factor, to get the corresponding side lengths in the second triangle. The scale factor is $\frac{15}{2} = 7.5$. The side lengths of the smaller triangle (which you found in Problem 5.2) are 2 units, 1 unit, and $\sqrt{3}$ units (or about 1.7 units). So, the side lengths of the larger triangle are 15 units, 7.5 units, and $7.5\sqrt{3}$ (or about 13.0 units).

c. The larger triangle's area is 7.5^2 (or 56.25) times the smaller triangle's area.

9. **a.** Since the circle has center (0, 0) and radius 13, the equation would be $(x - 0)^2 + (y - 0)^2 = 13^2$, or $x^2 + y^2 = 169$.

b. (0, 13) or (0, –13)

(5, 12) or (5, –12)

$(-4, \sqrt{153})$ or $(-4, -\sqrt{153})$

$(-8, \sqrt{105})$ or $(-8, -\sqrt{105})$

$(\sqrt{69}, 10)$ or $(-\sqrt{69}, 10)$

$(\sqrt{133}, -6)$ or $(-\sqrt{133}, -6)$

(13, 0) or (–13, 0)

$(\sqrt{165}, -2)$ or $(-\sqrt{165}, -2)$

10. **a.** Since the circle has center (0, 0) and radius 10, the equation of the circle is $(x - 0)^2 + (y - 0)^2 = 10^2$ or $x^2 + y^2 = 100$.

b. (8, 6) or (8, –6)

$(3, \sqrt{91})$ or $(3, -\sqrt{91})$

$(-4, \sqrt{84})$ or $(-4, -\sqrt{84})$

(0, 10) or (0, –10)

$(\sqrt{84}, -4)$ or $(-\sqrt{84}, -4)$

(8, –6) or (–8, –6)

(10, 0) or (–10, 0)

$(\sqrt{96}, 2)$ or $(-\sqrt{96}, 2)$

11. **a.** Yes. With the wireless router positioned at (0, 0), the tree house would be $\sqrt{600^2 + 800^2}$ ft (or 1,000 ft) away from the wireless router. Jada would be in range to get an Internet connection from her tree house.

b. $x^2 + y^2 = 1{,}400^2$ or $x^2 + y^2 = 1{,}960{,}000$

Connections

12. **a.** i, ii, iii, iv, v, vi

b. i, ii, iii, iv, v

c. i, iii, iv, vi

d. i, iii, iv

13. ≈ 4.8 cm. The diagonal of the large square is 6.8 cm (the sum of 4 segments of length 1.7 cm). Given that the sides of the square are the same length:

$$s^2 + s^2 = 6.8^2$$
$$2s^2 = 46.24$$
$$s^2 = 23.12$$
$$s = \sqrt{23.12} \approx 4.8$$

14. See Figure 1. The distance between the cars increases by 78.1 mi each hour. (**Note:** Students will probably calculate the distance apart by adding the sum of the squares and taking the square root of that sum.)

15. After 2 h, the northbound car has traveled 80 mi. Use this distance as one leg of a right triangle and the distance apart (100 mi) as the hypotenuse. Using the Pythagorean Theorem, $100^2 - 80^2 = 3{,}600$, so the distance the eastbound car has traveled must be $\sqrt{3{,}600} = 60$ mi. This distance was traveled in 2 h, so the eastbound car is traveling at 30 mph. (**Note:** This is a 3–4–5 right triangle with a scale factor of 20.

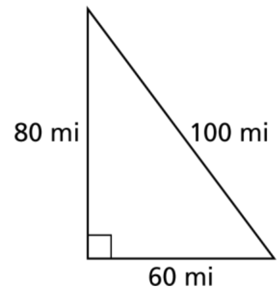


Figure 1

Distances Traveled by Two Cars

Hours	Distance Traveled by Northland Car (mi)	Distance Traveled by Eastbound Car (mi)	Distance Between Cars (mi)
1	60	50	$\sqrt{60^2+50^2} \approx 78.1$
2	120	100	$\sqrt{120^2+100^2} \approx 156.2$
3	180	150	$\sqrt{180^2+150^2} \approx 234.3$
4	240	200	$\sqrt{240^2+200^2} \approx 312.4$
n	$60n$	$50n$	$78.1n$

16. **a.** They are congruent.

b. 45°, 45°, and 90°. The diagonal divides the corner angles into two angles of equal measure, so the measures of the smaller angles must each be half of 90°, or 45°. Some students may use a protractor or angle ruler.

c. The legs of the right triangle each have a length of 1 unit, and $1^2 + 1^2 = 2$. So, the diagonal—which is the hypotenuse of a right triangle—has a length of $\sqrt{2}$ units.

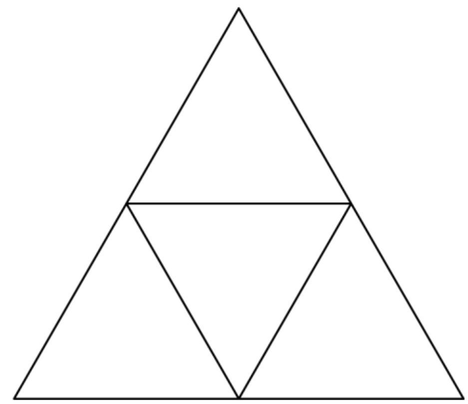
d. The measures of the angles would still be 45°, 45°, and 90°. Because $5^2 + 5^2 = 50$, the length of the diagonal would be $\sqrt{50}$ units. (**Note:** Some students may notice that $\sqrt{50} = \sqrt{25 \bullet 2} = 5\sqrt{2}$ or that this square is larger than the original by a scale factor of 5; thus, the diagonal must be 5 times as long, or $5\sqrt{2}$ units.)

17. **a.** All 45–45–90 triangles are similar to each other. If corresponding angles of a triangle have equal measure, then the triangles are similar.

b. The other leg must also be 5 units long because 45–45–90 triangles are isosceles. Applying the Pythagorean Theorem, you get $(\text{hypotenuse})^2 = 5^2 + 5^2 = 50$, so the length of the hypotenuse $= \sqrt{50} = 5\sqrt{2} \approx 7.07$ units. So, the perimeter is $5 + 5 + 5\sqrt{2} \approx 17.07$ units.

18. 1,012.4 m. The first segment along the ground is the leg of an isosceles right triangle. Because the other leg is 15 m long, this leg also has a length of 15 m. The same argument holds for the last segment along the ground. Therefore, the horizontal portion of the cable is $1{,}000 - (2 \cdot 15) = 970$ m long. Each angled part of the cable is the hypotenuse of an isosceles right triangle with legs of length 15 units. Because $15^2 + 15^2 = 450$, each angled piece has length $\sqrt{450} \approx$ 21.2 m. The overall length of the cable is thus $970 + 21.2 + 21.2 \approx 1{,}012.4$ m.

19. **a.** Yes.



b. The length of a side of the larger equilateral triangle is 3, so the height is $1\frac{1}{2} \bullet \sqrt{3} \approx 2.60$ inches.

c. Yes. If Pat cuts each smaller equilateral triangle in half, then he can reposition the halves to make a small rectangle with dimensions 0.75 inch by approximately 1.30 inches (or $\frac{3\sqrt{3}}{4}$ inches to be exact). Since there are four of these newly transformed rectangles, Pat can place them in a 1-by–4 arrangement (which, depending on orientation, would produce a larger rectangle with dimensions 0.75 inch by 5.20 inches or a larger rectangle with dimensions 3 inches by 1.30 inches) or in a 2-by–2 arrangement (with dimensions 1.50 inches by 2.60 inches).

20. –6

21. –28

22. –105

23. –11

24. 114

25. –42

26. 69

27. –65

28. –240

29. –4

30. 21

31. –21

32. $y = \frac{1}{2}x + 3$

33. $y = -\frac{1}{3}x + 5$

34. $y = 6x + \frac{1}{2}$

35. $y = 2x - 5$

36. $y = -4x + 3$

37. $y = -\frac{5}{6}x + 5$

38. **a.** The side length of the larger square is $6\sqrt{2}$ or approximately 8.5 meters. The area of the larger square is 72 meters^2. The radius of the circle is 6 meters, so the radius of the square is also 6 meters. If you draw two radii of the square, you get a 45–45–90 triangle with leg lengths of 6 meters. Use either the Pythagorean Theorem or ratio rules to find that the hypotenuse, which is also the side length of the larger square, is $6\sqrt{2}$ meters. So, the area of the larger square is $(6\sqrt{2})^2 = 72$ meters^2.

b. 36 meters^2. Given the answer in part (a), half of the length of the side of the larger square is also the length of a leg in the smaller gray 45–45–90 triangle. Use either the Pythagorean Theorem or ratio rules to find that the hypotenuse of the gray triangle, which is also the side length of the smaller square, is 6 meters. Squaring this value gives the area of the smaller square to be 36 meters^2.

c. 36 meters^2. Use your answers from parts (a) and (b). Subtract the area of the smaller square, 36 meters^2, from the area of the larger square, 72 meters^2. (**Note:** Students may notice that they can move the gray 45–45–90 triangles (the area between the larger and smaller squares) inside of the smaller square, so that the area of the region between the two squares is equal to the area of the smaller square.)

d. ≈ 41.1 meters^2. Given that the radius of the circle is 6 meters, the area of the circle is 36π. From part (a), you know that the area of the larger square is 72 meters^2. So, the difference between the two areas is $36\pi - 72$ meters^2, or approximately 41.1 meters^2.

39. **a.** Given the center at (0, 0) and a radius of 6 units, the equation would be $x^2 + y^2 = 36$.

b. $LS = 6\sqrt{2}$ units. Area of $LSJK$ = 72 units^2. Similar reasoning as Exercise 38, part (a).

c. $PR = 6$. Area of $PRVT$ = 36 units^2. Half of the length of a side of $LSJK$, say LS, is also the length of a leg in a 45–45–90 triangle, say, triangle PSR. Use either the Pythagorean Theorem or ratio rules to find that the hypotenuse of triangle PSR, which is also the side length of $PRVT$, is 6 units. Squaring this value gives the area of $PRVT$ to be 36 units^2.

d. $P(-3, 3)$, $R(3, 3)$. The length of PR is 6, as found in part (c). Point P is a reflection of point R over the y-axis, so the distance from the y-axis to either point is 3. Similar reasoning to find the distance both points are above the x-axis.

Extensions

40. **a.** ≈ 39.1 miles. The distance between the store, and the library is 25 miles and the distance from the library to the movie theater is 30 miles. The three roads are the legs of a right triangle. Using the Pythagorean Theorem, the distance from the store to the movie theater is $\sqrt{25^2 + 30^2} \approx 39.1$ miles.

b. 0.71 hours. Using $d = rt$, with $d = 39.1$ miles and $r = 55$ mph, $t \approx 0.71$ hours, or about 42.7 minutes.

41. **a.** (3.54, 3.54). If you draw a segment from point B to the x-axis, you get a 45–45–90 triangle with a hypotenuse of 5. Therefore, the length of each leg would be $\frac{5}{\sqrt{2}}$ units, or approximately 3.54 units. So, the coordinates of point B are (3.54, 3.54).

b. 1. The rise is $\frac{5}{\sqrt{2}}$ and the run is $\frac{5}{\sqrt{2}}$, so the ratio of rise to run is equal to 1.

42. **a.** 15 cm. One side of the equilateral triangle is the same as a side of the square paper.

b. ≈ 13.0 cm. If you fold the equilateral triangle in half, you get two 30–60–90 triangles. The shorter leg of one of the triangles (the side opposite the 30 degree angle) is half of a side of the equilateral triangle, or 7.5 cm. The height of the equilateral triangle would be equal to the length of the longer leg of the 30–60–90 triangle, which is $\sqrt{3}$ times longer than the shorter leg. So, the height is $7.5\sqrt{3} \approx 13.0$ cm.

43. **a.** *AB* = 48 units, *BD* = 36 units. To find *BD*, use triangle *BCD* as a 30–60–90 triangle. Given *CD*, multiply by $\sqrt{3}$ to find that *BD* = 36. To find *AB*, add *AD* and *BD*, so 12 + 36 = 48.

b. $24\sqrt{3} \approx 41.57$ units. It is twice the length of *CD*.

c. Ky is using side *AC* as the base of the triangle. So, the base is 24. The height would then be the length of *BC*.

d. Mario is using side *AB* as the base of the triangle. So, the base is 48. The height would then be the length of *CD*.

e. Jen is breaking up the larger triangle into two smaller triangles, finding the area of each smaller triangle, and combining those areas.

f. Yes. They are all using the area formula for a triangle, but using different bases and heights. Ky and Mario choose different sides for the base, which, in turn, makes them use different heights. Jen's way works because there is no overlap and no gap between the two smaller triangles.

44. **a.** *P*(–3, 3), *R*(3, 3), *V*(3, –3), *T*(–3, –3). Each of these points is the midpoint of the sides of the larger square.

b. $y = x$. The line has slope 1 and *y*-intercept 0.

c. *W*(4.24, 4.24), *Z*(–4.24, –4.24). The distance from the origin to point *W* is 6 units, the radius of the circle. If you draw a segment from *W* to the *x*-axis, you get a 45–45–90 triangle with side lengths of $\frac{6}{\sqrt{2}}$ units, which is approximately equal to 4.24 units. So, *W* is located at approximately (4.24, 4.24). Similar reasoning for point *Z*, but both values are negative because *Z* is in the third quadrant. Both points are on the circle because when you substitute their *x*- and *y*-values into the equation of the circle, $x^2 + y^2 = 36$, they satisfy the equation. (**Note:** If you use the rounded values for the coordinates of *W* and *Z*, the value on the left side of the equation will be close to, but not exactly equal to, 36.)

d. *M*(–4.24, 4.24), *N*(4.24, –4.24). Similar reasoning as in part (c).

e. *WMZN* is a square. The lengths of each side are approximately 8.48 units, so all four sides are congruent. Sides *MW* and *ZN* are horizontal. (Their slopes are zero.) *MZ* and *WN* are vertical. (Their slopes are undefined.) So, any two adjacent sides are perpendicular.

45. **a.** (4, 6) and (–2, 6)

$$(4 - 1)^2 + (6 - 2)^2 = 25$$
$$3^2 + 4^2 = 25$$
$$9 + 16 = 25$$
$$25 = 25$$
$$(-2 - 1)^2 + (6 - 2)^2 = 25$$
$$(-3)^2 + 4^2 = 25$$
$$9 + 16 = 25$$
$$25 = 25$$

b. (5, 5) and (5, –1)

$(5-1)^2 + (5-2)^2 = 25$

$4^2 + 3^2 = 25$

$16 + 9 = 25$

$25 = 25$

$(5-1)^2 + (-1-2)^2 = 25$

$4^2 + (-3)^2 = 25$

$16 + 9 = 25$

$25 = 25$

c. (–3, 5) and (–3, –1)

$(-3-1)^2 + (5-2)^2 = 25$

$(-4)^2 + 3^2 = 25$

$16 + 9 = 25$

$25 = 25$

$(-3-1)^2 + (-1-2)^2 = 25$

$(-4)^2 + (-3)^2 = 25$

$16 + 9 = 25$

$25 = 25$

d. (1, 7) and (1, –3)

$(1-1)^2 + (7-2)^2 = 25$

$0^2 + 5^2 = 25$

$0 + 25 = 25$

$25 = 25$

$(1-1)^2 + (-3-2)^2 = 25$

$0^2 + (-5)^2 = 25$

$0 + 25 = 25$

$25 = 25$

e. (6, 2) and (–4, 2)

$(6-1)^2 + (2-2)^2 = 25$

$5^2 + 0^2 = 25$

$25 + 0 = 25$

$25 = 25$

$(-4-1)^2 + (2-2)^2 = 25$

$(-5)^2 + 0^2 = 25$

$25 + 0 = 25$

$25 = 25$

f. (4, 6) and (4, –2)

$(4-1)^2 + (6-2)^2 = 25$

$3^2 + 4^2 = 25$

$9 + 16 = 25$

$25 = 25$

$(4-1)^2 + (-2-2)^2 = 25$

$3^2 + (-4)^2 = 25$

$9 + 16 = 25$

$25 = 25$

46. a. Lengths of $\overline{AC}$, $\overline{BC}$, $\overline{AB}$ are (x – 1), (y – 2), and 5, respectively.

b. The equation $(x-1)^2 + (y-2)^2 = 25$ shows how the side lengths are related.

c. If students choose B as a point in another quadrant (not Quadrant I) they will meet the problem of how to calculate or name the length of each side of the triangle. They may reason from a specific example, say (–2, 6). Then the horizontal distance on the triangle is not "x – 1" but "1 – x" or "$|x-1|$" However, the equation involves "$(x-1)^2$" and so students should notice that, for example, $(-2-1)^2 = [1-(-2)]^2$. So, yes, points in other quadrants satisfy the equation.

d. $(x-m)^2 + (y-n)^2 = r^2$